ثلاثة مدني

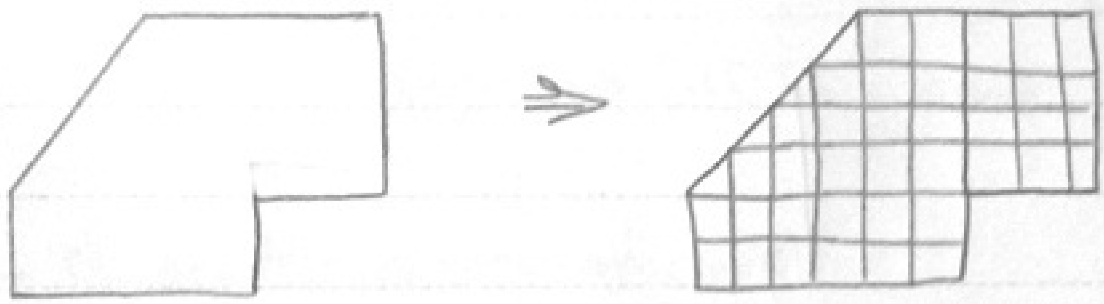
٢٠١٣-٢٠١٤

Finite Element

٣.٣ رمضان

with F.E.M.

$A = \pi r^2$ $\Rightarrow$ $A = \sum \left(\frac{1}{2} R^2 \sin\theta \right)$



with F.E.M.

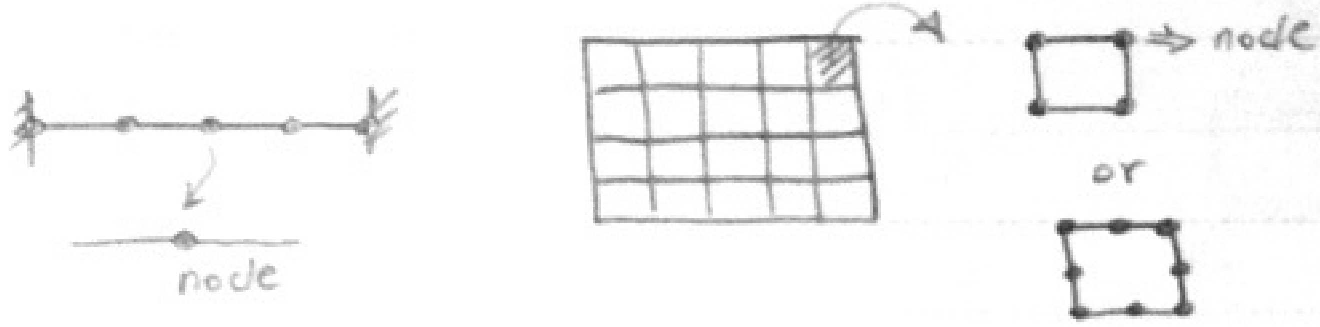
* يتم تقسيم الشكل الاصلى الى اشكال أصغر لنعطى نتائج اقرب الى الحقيقة
* حيث يصعب حل الشكل الاصلى فيقسم ليعطى حل مبسط وقريب من القيمة الحقيقية
* كلما زادت عدد الاجزاء كلما اقترب الحل من الواقع

* Feilds used F.E.M. "finite element method"

1- Mechanical engineering
2- structural analysis
3- Thermal fluid Mechanics
4- Electro magnetics
5- geo mechanics
6- Bio mechanics

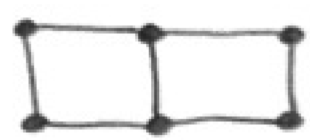
* steps of F.E.M.

1- Divid structure into pieces
"elements with nodes"



كلما زادت عدد node كلما كان الحل اقرب للحقيقة

2 - Connect elements of node



مشتركة بين 2 element

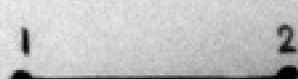
(ربط element ببعض)

3 - Describe behavior of physical quantity of each element
تحديد خواص وسلوك كل element

4 - solve system of equations "stiffness method"

* Types of element

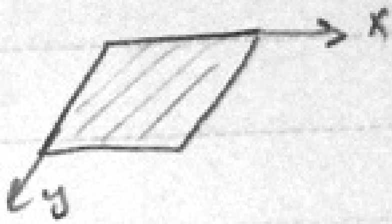
① 1-D element



<u>ex</u> spring, Truss, beam, pipe

(2) 2-D element

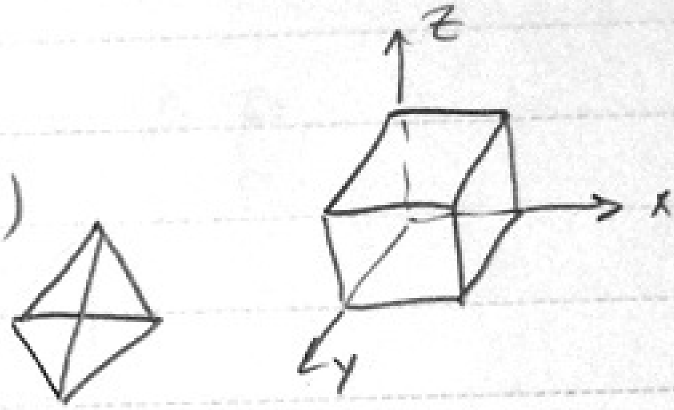
ex membrane, plate, shell



(3) 3-D element

ex 8-node element (solid)

6-node element (prism)



* Conditions To apply F.E.M

1- static loads "slow steady-rate of loading

تحميل ثابت وبسرعة بطيئة

2- elastic material

لم تصل الى Yeild

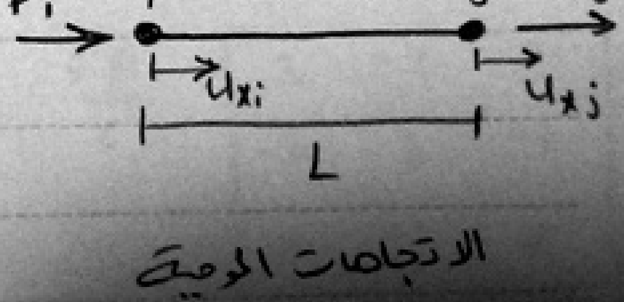
3- small deformation

Truss / Bar element

- 2 node $= i, j$
- each node has 1 Dof u_x

$\therefore$ we have u_{x_i}, u_{x_j}

- axial force element



الاتجاهات الموجبة

strain - disp. $\Rightarrow$ $\varepsilon = \frac{du}{dx} = \frac{\Delta}{L}$

stress_disp $\Rightarrow \quad \sigma = E\,\varepsilon = \dfrac{E\,\Delta}{L}$

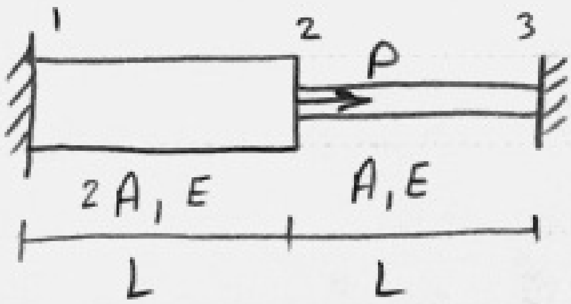
$$\because \sigma = \frac{F}{A}$$

$$\therefore F = \frac{EA}{L}\cdot\Delta = K\,\Delta$$

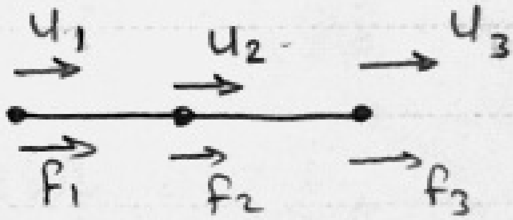
حيث $\quad K = \begin{bmatrix} \dfrac{EA}{L} & -\dfrac{EA}{L} \\ -\dfrac{EA}{L} & \dfrac{EA}{L} \end{bmatrix}$

ن $\quad \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \dfrac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$

Ex $\underline{\underline{1}}$



Find stress



sol

element $\underline{1}$

$$k_1 = \frac{2EA}{L}\begin{array}{c} \begin{array}{cc} ① & ② \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} ① \\ ② \end{array}$$

element $\underline{2}$

$$k_2 = \frac{EA}{L}\begin{array}{c} \begin{array}{cc} ② & ③ \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}\begin{array}{c} ② \\ ③ \end{array}$$

$$K_{tot.} = \frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$\therefore\ u_1 = u_3 = \text{zero}$

$f_2 = P$

$$\therefore\ P = \frac{EA}{L}\left[-2*0 + 3\,u_2 - 1*0\right] = \frac{3EA}{L}\,u_2$$

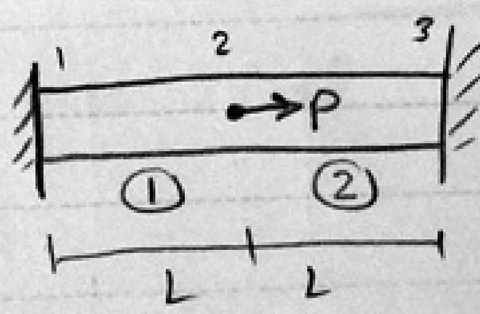
$$u_2 = \frac{PL}{3EA}$$

$$\therefore\ u = \frac{PL}{3EA}\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \Rightarrow \sigma = \frac{E\Delta}{L}$$

$$\sigma_1 = 2E * \frac{u_2 - u_1}{L} = 2E * \frac{\frac{P\cancel{L}}{3EA} - 0}{\cancel{L}} = \frac{2P}{3A}$$

$$\sigma_2 = E * \frac{u_3 - u_2}{L} = E * \frac{0 - \frac{PL}{3EA}}{L} = -\frac{P}{3A}$$

EX 2



$E = 2*10^4\ N/mm^2$

$A = 250\ mm^2$

$P = 6*10^4\ N$

$L = 150\ mm$

$\Delta_3 = 1.2\ mm$

find support reaction

$$k_1 = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad k_2 = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$\because\ u_3 = 1.2 \quad , u_1 = 0$

$f_2 = P = 6 * 10^4$

$$\therefore\ 6 * 10^4 = \frac{2 * 10^4 * 250}{150}\left(-1 * 0 + 2u_2 - 1 * 1.2\right)$$

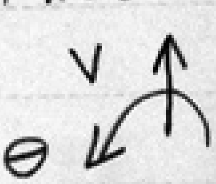
$u_2 = 1.5 \text{ mm}$

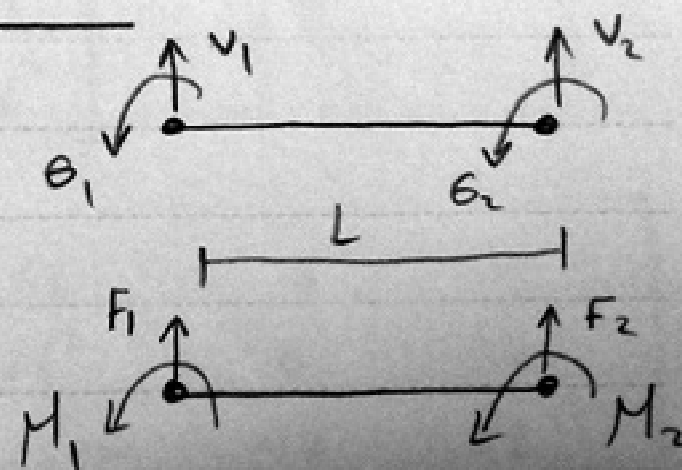
$f_1 = -5 * 10^4$ بالتعويض

$f_3 = -1 * 10^4$

Beam element

- each node have 2 DoF





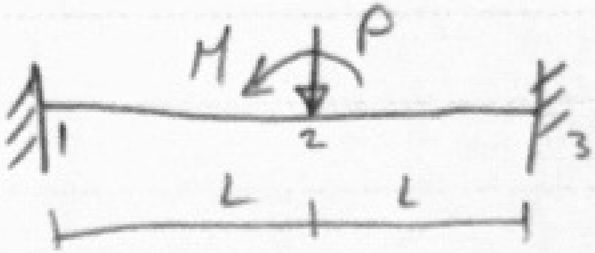
$$M = EI\,\frac{d^2v}{dx^2}$$

$$\sigma = \frac{-My}{I}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ M_1 \\ f_2 \\ M_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

Ex 3



Find deflection & rotation at center node
reactions at ends

$$K_1 = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$
$$K_2 =$$

$$\begin{array}{c c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} \quad & \quad & \quad & \quad \end{bmatrix} \end{array} \qquad \begin{array}{c c} & \begin{matrix}3 & 4 & 5 & 6\end{matrix} \\ \begin{matrix}3\\4\\5\\6\end{matrix} & \begin{bmatrix} \quad & \quad & \quad & \quad \end{bmatrix} \end{array}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

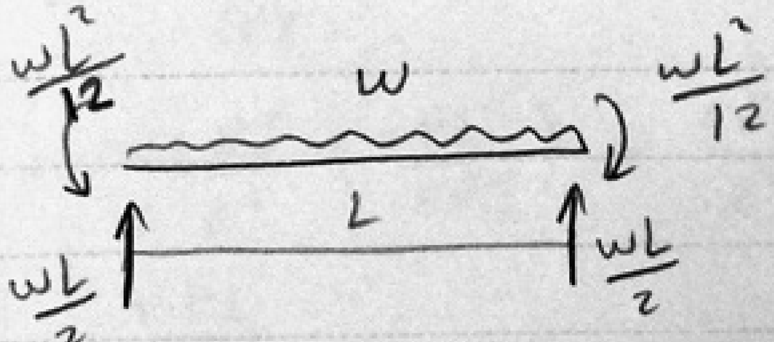
$$\begin{Bmatrix} F_1 \\ M_1 \\ \cancel{F_2} \\ \cancel{M_2} \\ F_3 \\ M_3 \end{Bmatrix} = \frac{EI}{L^3} \qquad \begin{Bmatrix} \cancel{v_1} \\ \cancel{\theta_1} \\ v_2 \\ \theta_2 \\ \cancel{v_3} \\ \cancel{\theta_3} \end{Bmatrix}$$

$v_1, \theta_1, v_3, \theta_3 = \text{zero}$ حيث

$M_2 = M$

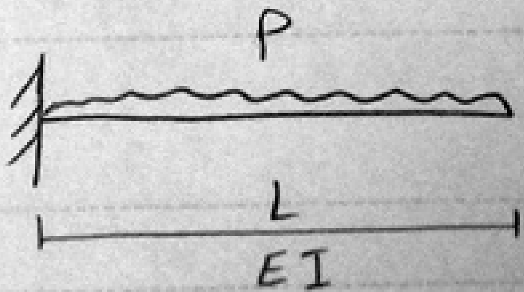
$F_2 = -P$

**



(fixed end moment)

<u>Ex 4</u>



Find deformation at free end

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ \frac{PL}{2} = \\ -\frac{PL^2}{12} = \end{array}\begin{Bmatrix} f_1 \\ M_1 \\ f_2 \\ M_2 \end{Bmatrix} = \qquad \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}\begin{array}{l} = 0 \\ = 0 \\ \\ \\ \end{array}$$

$$+\frac{12EI}{L^3} v_2 - \frac{6EI}{L^2}\theta_2 = \frac{+PL}{2}$$

$$-\frac{6EI}{L^2} v_2 + \frac{4EI}{L}\theta_2 = \frac{-PL^2}{12}$$

$$\therefore \theta_2 = -\frac{PL^3}{6EI} \qquad v_2 = -\frac{PL^4}{8EI}$$

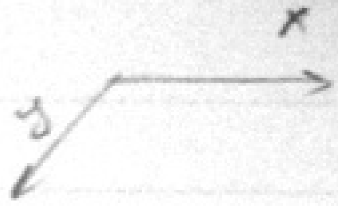
$$\therefore f_1 = -\frac{12EI}{L^3}\left(-\frac{PL^4}{8EI}\right) + \frac{6EI}{L^2}\left(-\frac{PL^3}{6EI}\right)$$

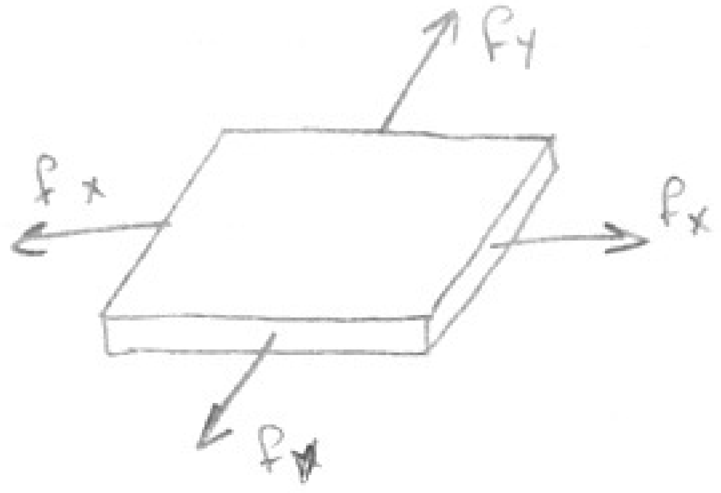
$$= +\frac{PL}{2}$$

$$M_1 = -\frac{6EI}{L^2}\left(-\frac{PL^4}{8EI}\right) + \frac{2EI}{L}\left(-\frac{PL^3}{6EI}\right)$$

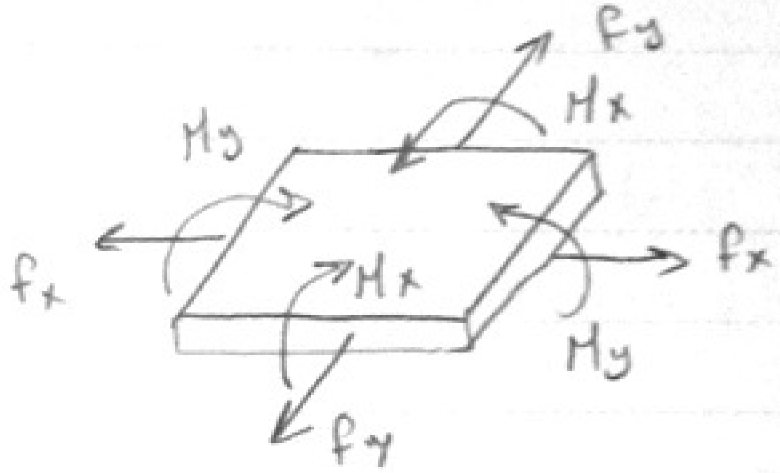
$$= \frac{PL^2}{12}$$

Shell element

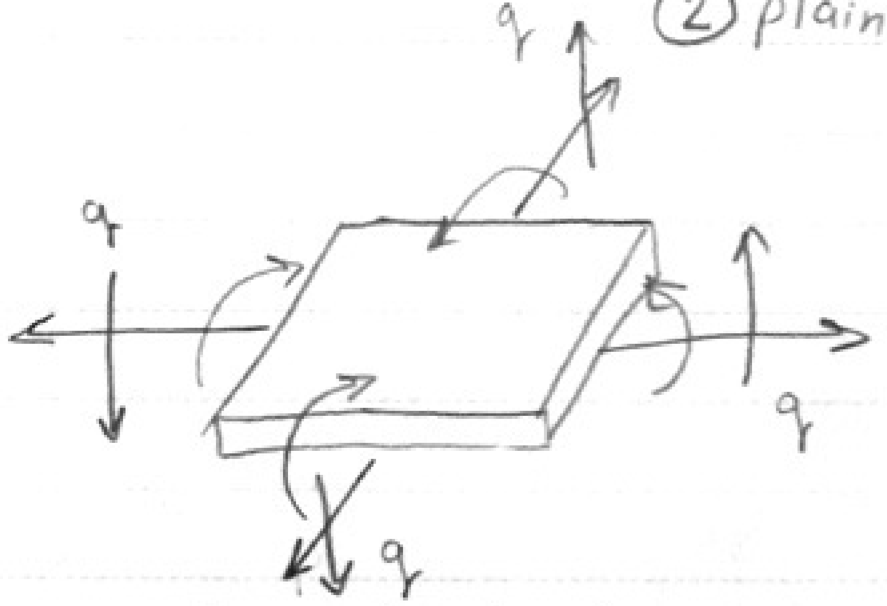




① Memberane

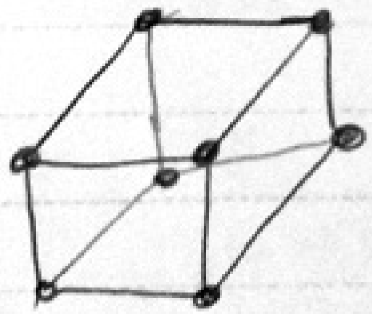


② plain stress



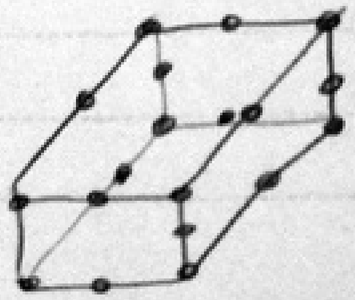
③ plate bending

solid element



① Hexahedral (Brick)
8 nodes
(linear)

② quadratic
20 nodes



strain - disp.

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$